Wave transport in complex media in the presence of disorder

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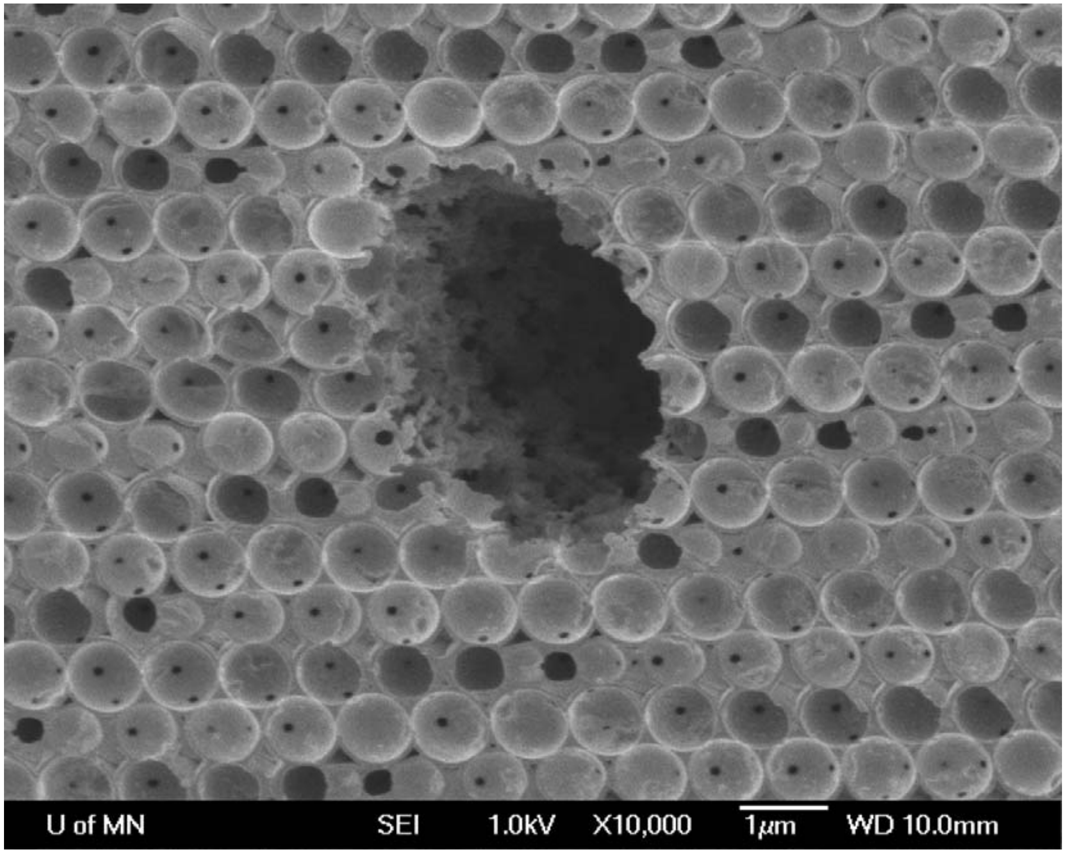
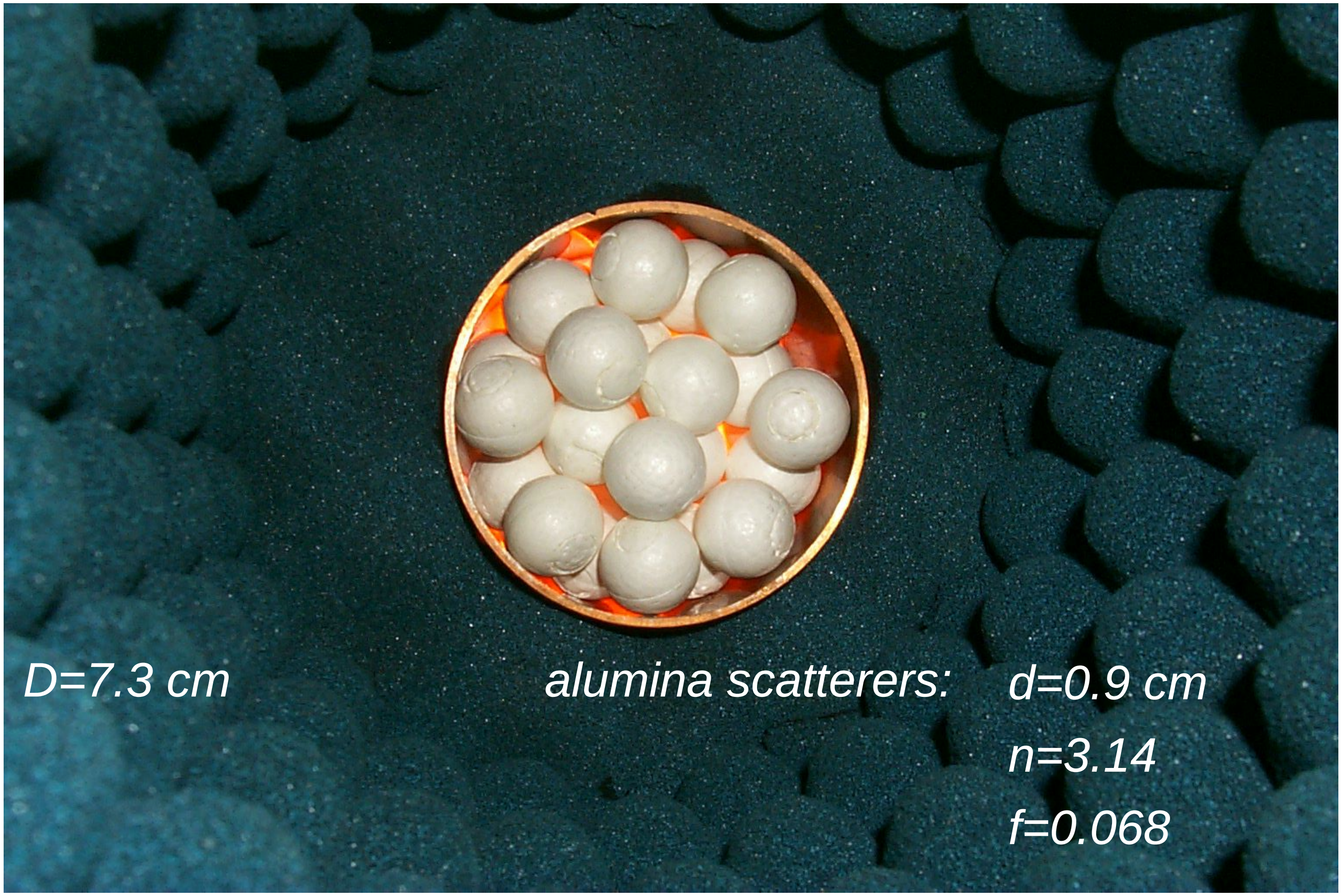
Intentional Linear Defect in Si Inverted Opal



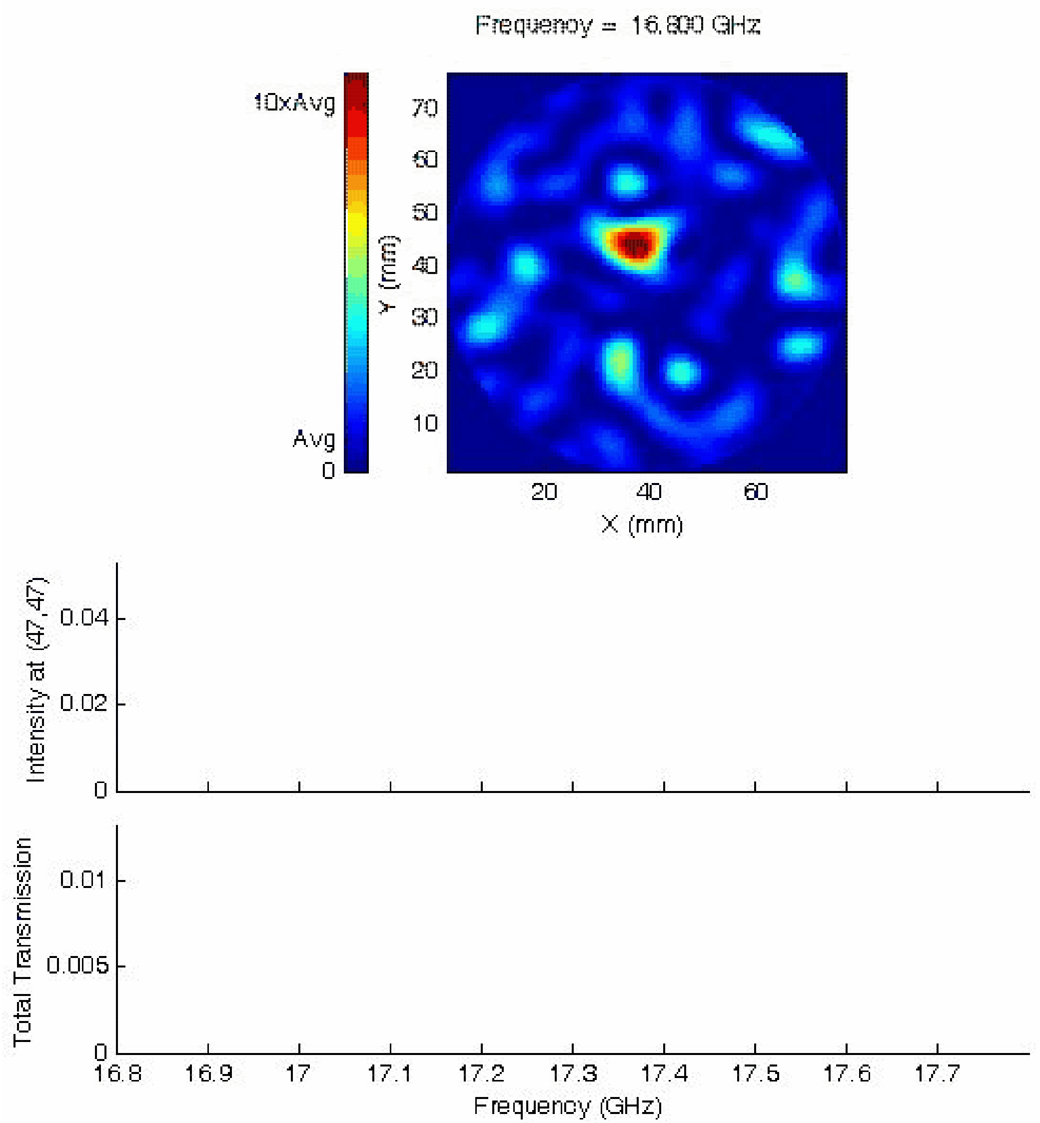
expect strong EM wave scattering from rough boundaries of the waveguide

Alumina Random Sample



to study transport in the presence of disorder, look at microscopic samples

Transmission in a disordered waveguide

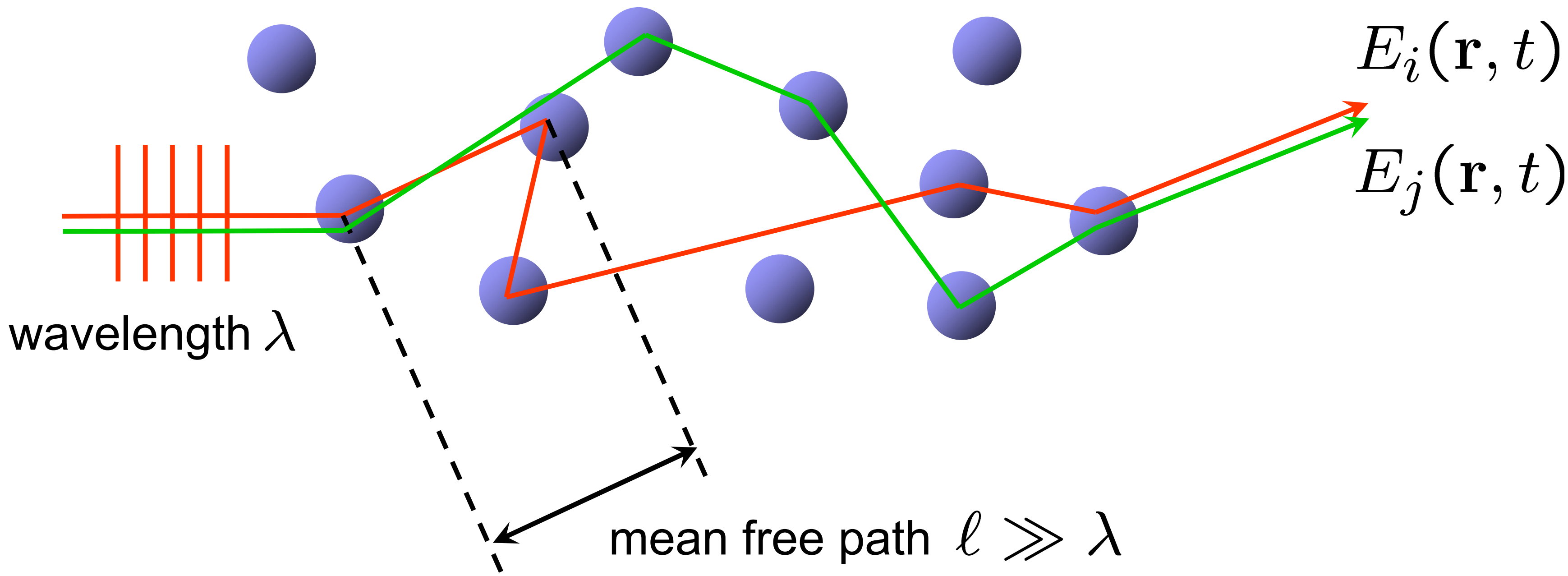


stationary sample, $L >> \ell$

D=7.5 cm

λ~*2 cm*

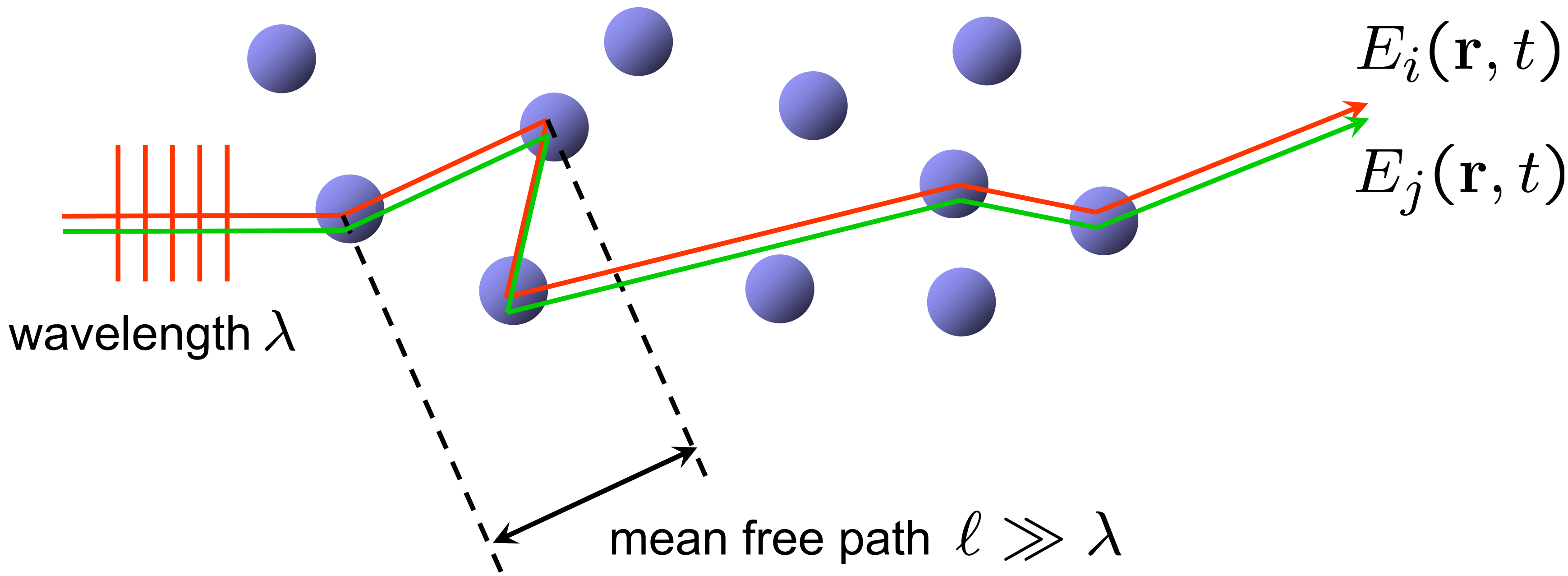
Wave propagation in a disordered medium



Field $E(\mathbf{r},t) = \sum_i E_i(\mathbf{r},t)$

Intensity $I(\mathbf{r},t) = |E(\mathbf{r},t)|^2 = \sum_{i,j} E_i(\mathbf{r},t) E_j^*(\mathbf{r},t)$

Wave diffusion in a disordered medium

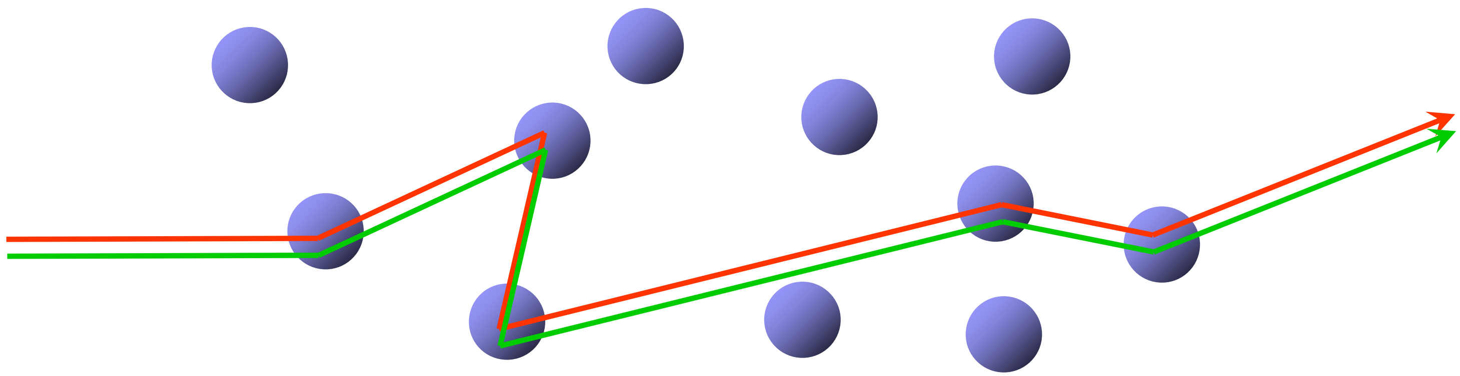


Average intensity:

$$\langle I(\mathbf{r},t)\rangle = \sum_i \left\langle |E_i(\mathbf{r},t)|^2 \right\rangle + \sum_{i\neq j} \left\langle E_i(\mathbf{r},t) E_j^*(\mathbf{r},t) \right\rangle$$

Only the pairs of **identical** paths have the same phase
and thus give a contribution to the average intensity

Wave diffusion in a disordered medium



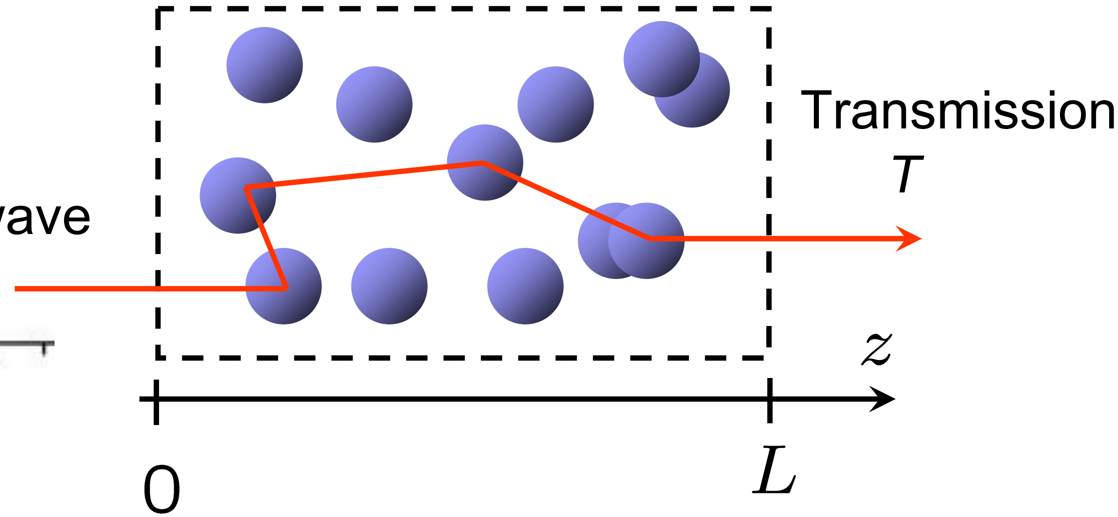
$$\ell \gg \lambda$$

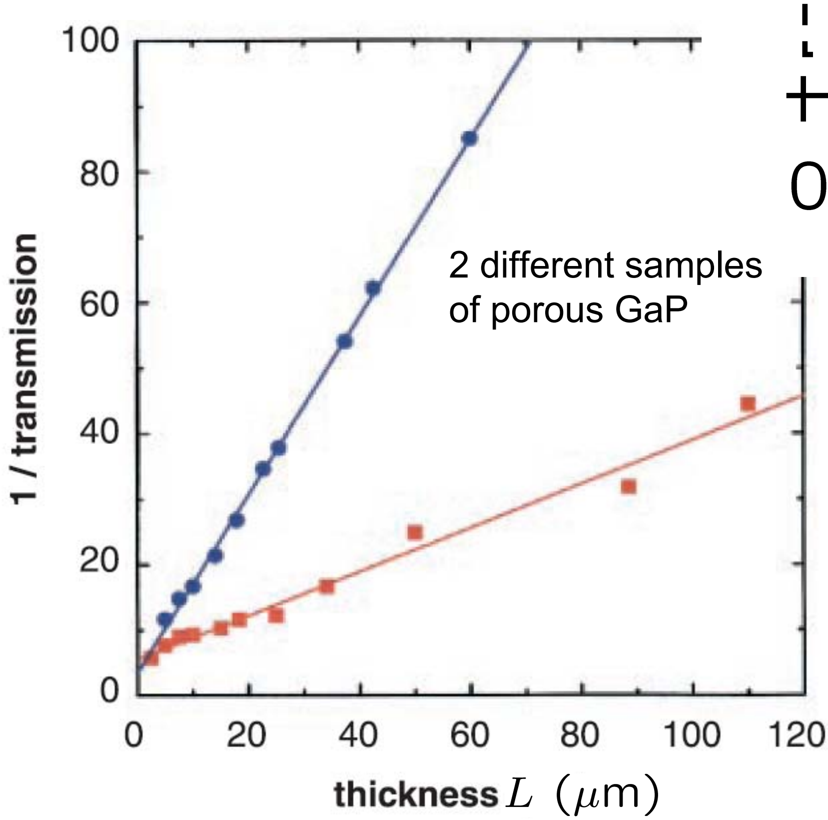
Diffusion equation for the average intensity:

$$\left[\frac{\partial}{\partial t} - D_B \nabla^2\right] \langle I(\mathbf{r}, t) \rangle = \text{Source}(\mathbf{r}, t)$$

(This equation would yield the Ohm's law for a disordered conductor)

Diffusion approximation works very well in 3D



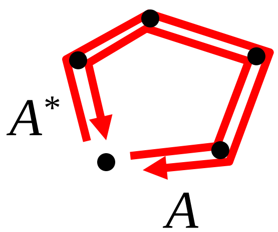


Ohm's law:

$$T \propto \frac{1}{L}$$

F.J.P. Schuurmans *et al.*, *Science* **284,** 141 (1999)

Coherent backscattering



Probability of return: $\left(A + A^*\right)^2 = 4A^2, \quad A^2 + A^{*2} = 2A^2$

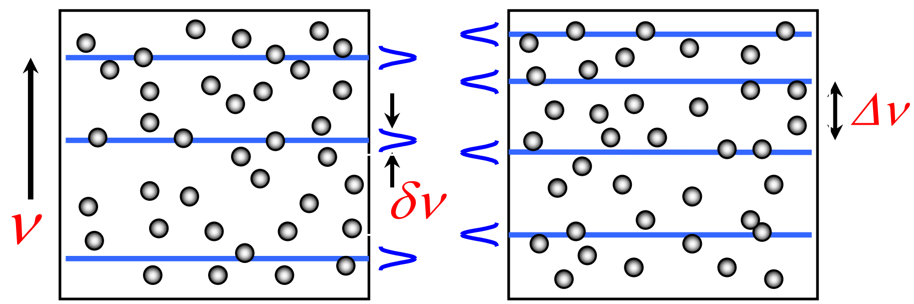
wave | particle

- transport reduction
- nonlocal correlation

⇒

- photon localization
- non-Gaussian statistics

Photon localization parameter



Linewidth: $\delta\nu \sim 1/\tau \sim D/L^2$ Level spacing: $\Delta\nu = 1/N(\nu)$

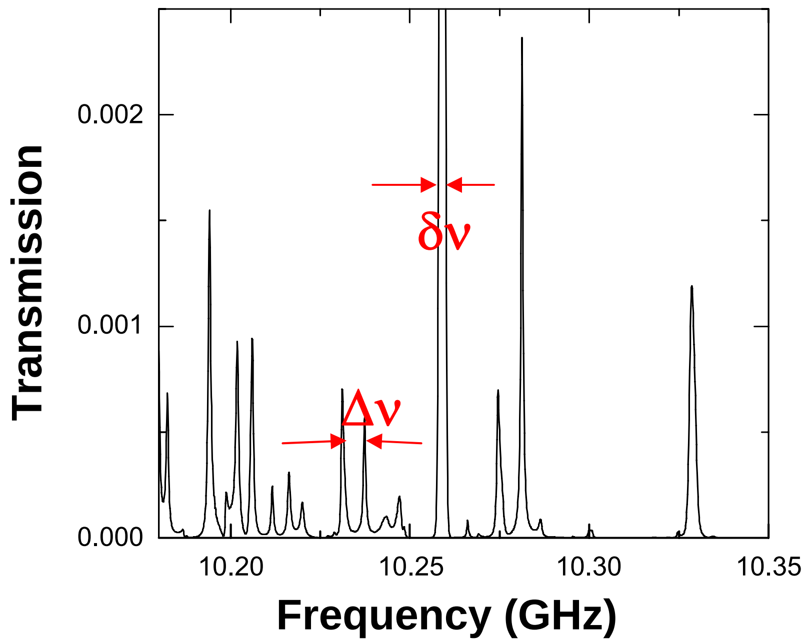
Number of modes within linewidth, Thouless number: $\delta = \delta\nu / \Delta\nu$

*Thouless, PRL **39**, 1167 (1977)*

In waveguide, $N(\nu) \sim L$, and therefore, $\delta = \delta\nu\, N(\nu) \sim 1/L$

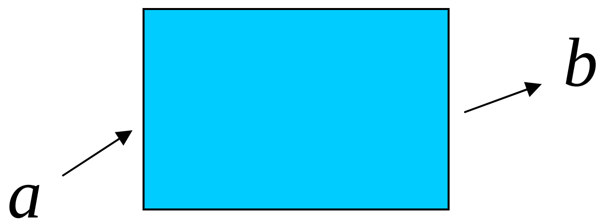
Photon localization

- for $\delta < 1$, levels in different blocks of material don't overlap
- coupling between modes and transport are suppressed
- wave is localized



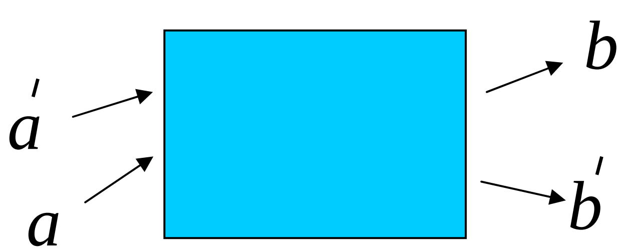
$$\delta = (dN / d\nu)(D / L^2) = G / (e^2 / h) = g$$, dimensionless conductance

Transmission coefficients



Intensity, $T_{ab} = |t_{ab}|^2,$ $\quad s_{ab} = T_{ab}/\langle T_{ab}\rangle$

Transmission, $T_a = \sum_b T_{ab},$ $\quad s_a = T_a/\langle T_a\rangle$



Transmittance, $T = \sum_a T_a = \sum_{ab} T_{ab},$

$$T = g \equiv G/(e^2/h)$$

Landauer, *Phil. Mag.* ***21****, 863 (1970)*

Statistics of transmission

In the absence of absorption, probability distributions of s_{ab}, s_a are determined by a single parameter, g

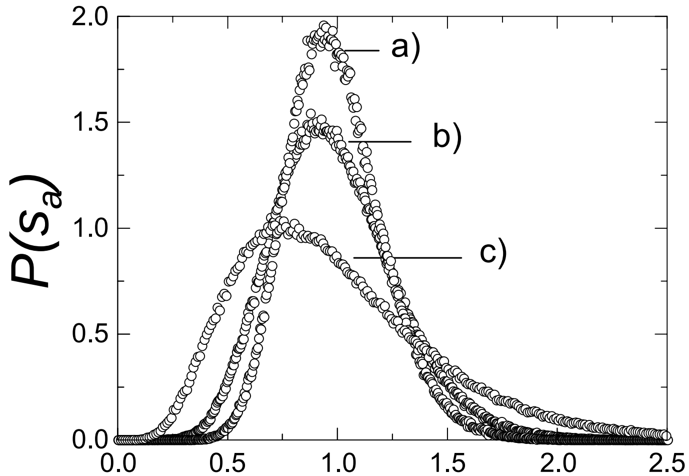
$$P(s_{ab}) = \int_{-i\infty}^{i\infty} \frac{ds_a}{s_a} P(s_a) \exp\left(-\frac{s_{ab}}{s_a}\right)$$

$$P(s_a) = \int_{-i\infty}^{i\infty} \frac{dx}{2\pi i} \exp(xs_a - \Phi(x)), \qquad \mathrm{var}(s_a) = 2/3g$$

$$\Phi(x) = g \ln^2\left(\sqrt{1 + x/g} + \sqrt{x/g}\right)$$

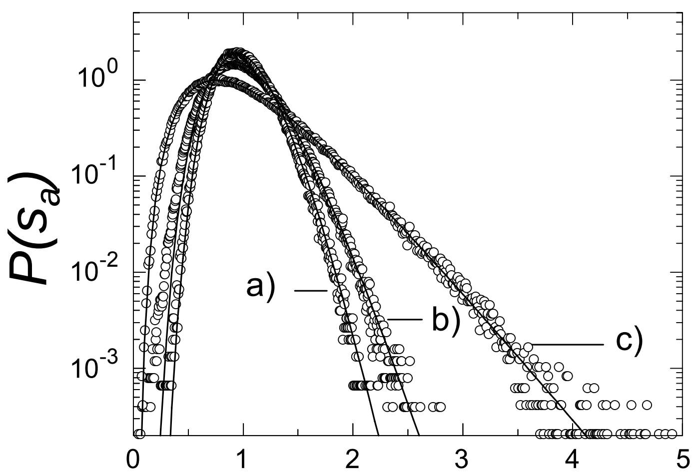
Van Rossum and Nieuwenhuizen, *PRL*,1994

Probability distributions of transmission



Fit: $g \rightarrow 2/3\,\mathrm{var}(s_a)$

var(s_a) is a localization parameter ?



Chabanov, Stoytchev, and Genack, *Nature* ***404****, 805 (2000)*

Pulse Propagation in Diffusive Medium

Breakdown of Diffusion

A: L = 61cm, g ≈ N*l*/L ≈ 7.4

$$t > t_D = (L+2z_0)^2/\pi^2 D_0$$

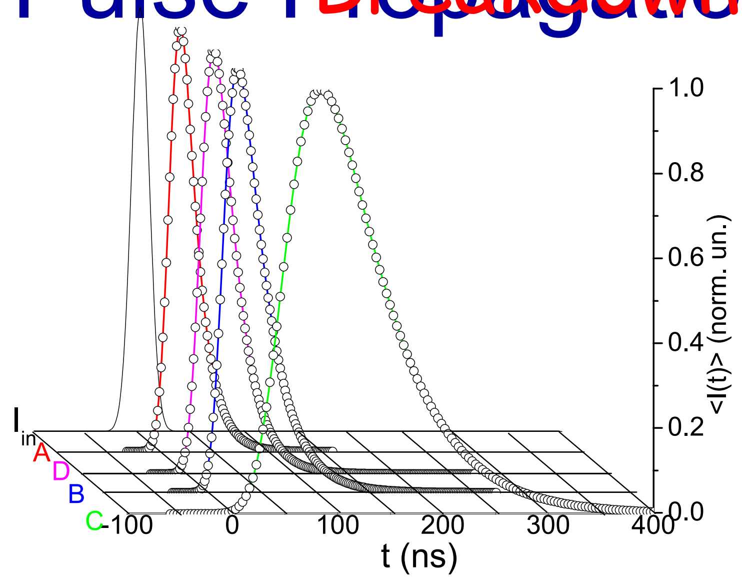
B: L = 90cm, g ≈ 5.1

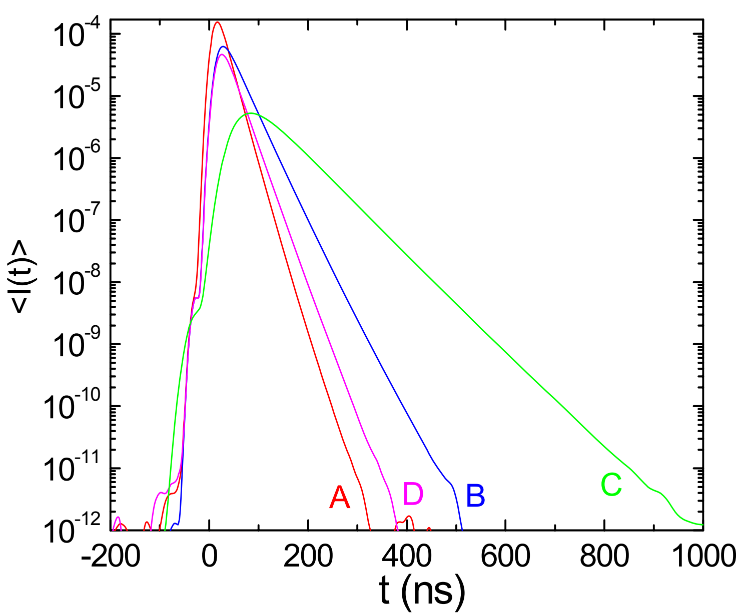
C: L = 183cm, g ≈ 2.7

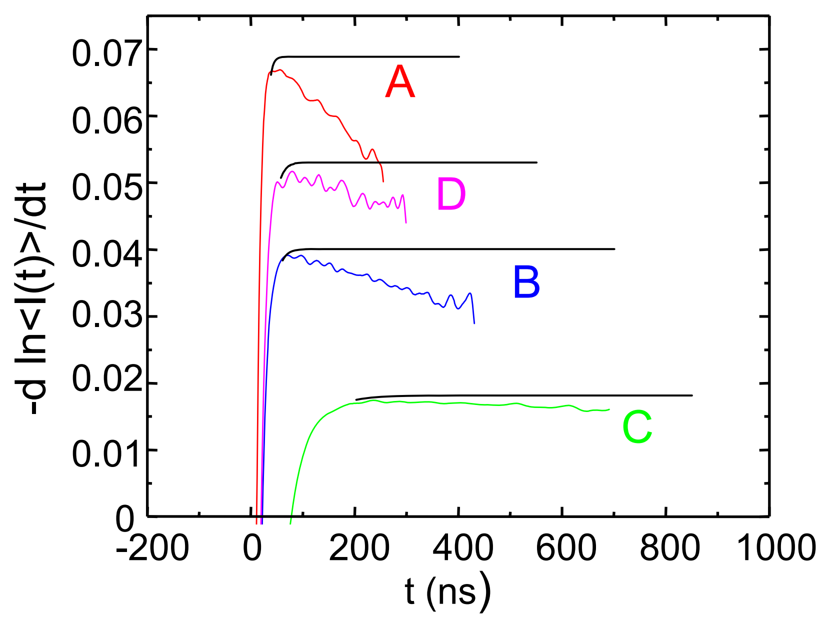
$$< I(t) > \to e^{-\gamma t} = e^{-(\frac{1}{t_D}+\frac{1}{\tau_a})t}$$

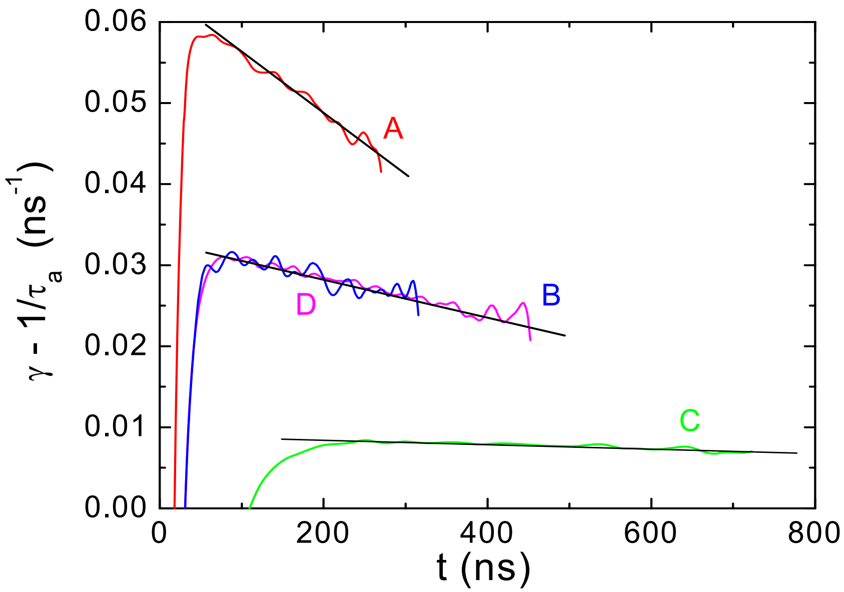
D: L = 90cm, added absorption,

$$D_0 = (\gamma - \frac{1}{\tau_a})(L+2z_0)^2/\pi^2$$









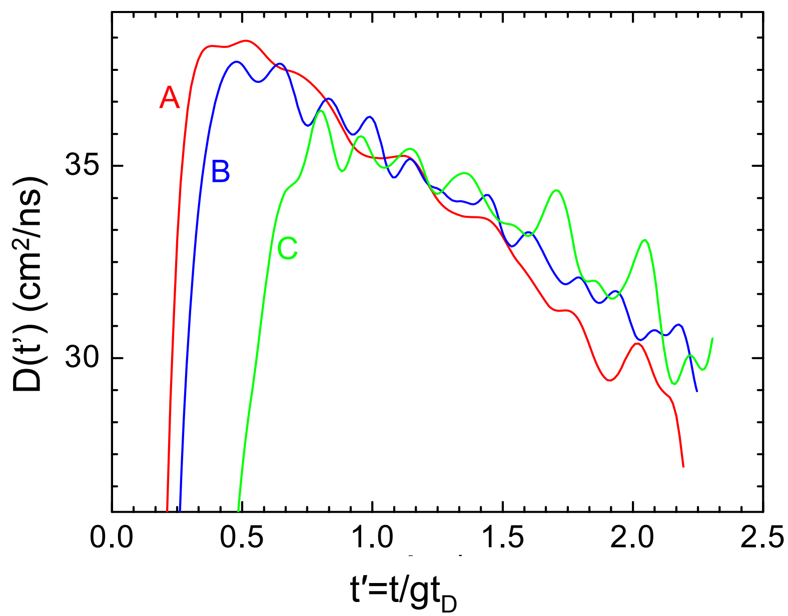
Non-exponential decay:

*A.D. Mirlin Phys. Rep. **326**, 259 (2000)*

Electron survival probability

$$< I(t) > \rightarrow \int P(\alpha) e^{-\alpha t} d\alpha$$

$$\approx e^{-(\frac{t}{t_D})(1-\frac{t}{2\pi^2 g t_D})} = e^{-\gamma(t)t}$$



time-dependent diffusion coefficient

$$D(t) = (\gamma(t) - \frac{1}{\tau_a})(L + 2z_0)^2 / \pi^2$$

$$= D_0(1 - t / 2\pi^2 g t_D)$$

*Chabanov et al. PRL, **90**, 203903 (2003)*

Summary

There are a number of parameters describing the extent of localization in disordered samples; however, in absorbing samples, only the variance of relative fluctuations of total transmission, var(s_a), is robust localization parameter.

There are a number of outstanding problems in the area of transport in random media awaiting for their solutions. These may greatly enhance our understanding of EM wave propagation in naturally existing and artificial media.

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1957 BCS theory of superconductivity

Superconductivity was first observed in 1911, but it is not explained until 1957, when John Bard

Theory of Superconductivity

[J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957)]

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1958 Anderson localization

Anderson lays the foundation for a quantum-mechanical theory of transport in systems with a certa

Absence of Diffusion in Certain Random Lattices

[P. W. Anderson, Phys. Rev. 109, 1492 (1958)]

Read Abstract | Read Paper

1958 First issue of *Physical Review Letters*

Volume 1, Issue 1 contains 25 Letters, including "Rest Mass of the Neutrino" by J. J. Sakurai, "Elemer

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