The Physical Nature of Materials Strengths**

By Zhe-Feng Zhang* and Jürgen Eckert

The strength of a material is assessed most often by means of a tensile test. For a given material with an original cross-section area \( A_0 \), if the applied maximum tensile force is equal to \( F_{\text{max}} \), the fracture strength can be calculated by \( \sigma_n = F_{\text{max}} / A_0 \) as described in the textbooks.\(^1,2\) For a bulk metallic glassy specimen, it often fails in a shear mode, as shown in Figure 1, and the shear fracture surface makes an angle of \( \theta_T = 56° \) with respect to the tension axis. Such shear fracture behavior has been widely observed in many metallic glasses, as summarized in the literatures\(^3,4\) and Table 1.\(^5–14\) According to the definition in the textbooks,\(^1,2\) the tensile fracture strength of the metallic glass should be equal to \( \sigma_n = F_{\text{max}} / A_0 \). However, the actual area of the shear fracture surface becomes \( A_0 / \sin(\theta_T) \) and the applied normal tensile force on the shear plane is \( F_{\text{max}} \cos(\theta_T) \), as shown in Figure 1. This will result in another tensile strength \( F_{\text{max}} \sin(\theta_T) \cos(\theta_T) / A_0 \), which is different from that \( (F_{\text{max}} / A_0) \) defined in textbooks.\(^1,2\) Consequently, this gives rise to some interesting and significant questions. Which is the real tensile strength of a metallic glass, \( F_{\text{max}} / A_0 \) or \( F_{\text{max}} \sin(\theta_T) \cos(\theta_T) / A_0 \)? Why do metallic glasses often fail neither along the maximum normal stress plane \( (\theta_T = 90°) \) nor along the maximum shear stress plane \( (\theta_T = 45°) \) under tensile loading?\(^5–14\) What is the physical nature of the materials strength?

For a material subjected to a tensile force \( F \), there is always a combined stress \( (\sigma_n, \tau_0) \) on any plane, as illustrated in Figure 2(a). In order to better understand the physical nature of materials strength and answer the interesting questions above, we proposed that there are only two independent intrinsic strengths \( \sigma_0 \) and \( \tau_0 \) for an isotropic material\(^4\). As illustrated in Figure 2(b), \( \sigma_0 \) is defined as the critical strength of a material in a Mode I failure; \( \tau_0 \) is the critical strength of a material in a Mode II fracture. If any plane of a material is subjected to a combined stress \( (\sigma_n, \tau_0) \), the tensile failure condition can be expressed by the following criterion:\(^4\)

\[
(\sigma_n / \sigma_0)^2 + (\tau_n / \tau_0)^2 = 1. \tag{1}
\]

Meanwhile, the tensile stress state \( (\sigma_n, \tau_0) \) on any shear plane follows the Mohr-circle equation, i.e.

\[
(\sigma_n - \sigma_T / 2)^2 + (\tau_n)^2 = (\sigma_T / 2)^2. \tag{2}
\]

According to Equations 1 and 2, the two independent strengths, \( \tau_0 \) and \( \sigma_0 \) can be derived as:

\[
\tau_0 = \sigma_T / 2\sqrt{1 - a^2}. \tag{3a}
\]

\[
\sigma_0 = \sigma_T / 2a\sqrt{1 - a^2}. \tag{3b}
\]

Here, \( \sigma_T = F_{\text{max}} / A_0 \) is the so-called tensile fracture strength; \( a \) is the fracture mode factor\(^4\) and can be calculated from the macroscopic tensile shear fracture angle, see the discussion below. Here, it is suggested that \( \sigma_T = F_{\text{max}} / A_0 \) can be only regarded as nominal fracture strength, rather than the intrin-
Table 1. Comparison of fracture strength, tensile shear fracture angles and intrinsic strengths for different metallic glasses from the references available. The fracture strength and tensile shear fracture angles were obtained by the different investigators for different metallic glasses, then the two intrinsic strengths and ratio were calculated according to the unified fracture criterion.

<table>
<thead>
<tr>
<th>Investigators</th>
<th>Compositions</th>
<th>( \sigma_T = \frac{F_{\text{max}}}{A_0} ) (GPa)</th>
<th>( \theta_T ) (degree)</th>
<th>( \tau_0 ) (GPa)</th>
<th>( \sigma_0 ) (GPa)</th>
<th>( \alpha = \tau_0 / \sigma_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>He et al. 5</td>
<td>Zr_{52.9}Ni_{14.9}Al_{19.7}Ti_{5}</td>
<td>1.66</td>
<td>-55</td>
<td>0.96</td>
<td>1.91</td>
<td>0.504</td>
</tr>
<tr>
<td>Inoue et al. 6</td>
<td>Cu_{40}Zr_{50}Ti_{10}</td>
<td>2.00</td>
<td>-54</td>
<td>1.14</td>
<td>2.36</td>
<td>0.485</td>
</tr>
<tr>
<td>Lewandowski et al. 7</td>
<td>Zr_{40}Ti_{12}Ni_{8.3}Cu_{12.1}Be_{22}</td>
<td>1.98</td>
<td>-51.6</td>
<td>1.11</td>
<td>2.44</td>
<td>0.455</td>
</tr>
<tr>
<td>Liu et al. 8</td>
<td>Zr_{52.5}Ni_{14.6}Al_{12.7}Ti_{5}</td>
<td>1.65</td>
<td>-54</td>
<td>0.94</td>
<td>1.95</td>
<td>0.485</td>
</tr>
<tr>
<td>Mukai et al. 9</td>
<td>Pd_{42}Ni_{49}P_{22}</td>
<td>1.65</td>
<td>-56</td>
<td>0.97</td>
<td>1.85</td>
<td>0.522</td>
</tr>
<tr>
<td>Noskova et al. 10</td>
<td>Co_{50}Si_{25}B_{30}Fe_{5}</td>
<td>1.48</td>
<td>-60</td>
<td>0.91</td>
<td>1.57</td>
<td>0.577</td>
</tr>
<tr>
<td>Xiao et al. 11</td>
<td>Zr_{52.5}Cu_{19}Ni_{3}Al_{15}Be_{22.7}</td>
<td>1.75</td>
<td>-55</td>
<td>1.01</td>
<td>1.96</td>
<td>0.504</td>
</tr>
<tr>
<td>Zhang et al. 12</td>
<td>Zr_{52.5}Ni_{14.6}Al_{10}Cu_{32.7}Ti_{5}</td>
<td>1.66</td>
<td>-56</td>
<td>0.97</td>
<td>1.86</td>
<td>0.522</td>
</tr>
<tr>
<td>Zhang et al. 13</td>
<td>Zr_{52.5}Ni_{14.6}Al_{10}Cu_{12.7}Ti_{5}</td>
<td>1.58</td>
<td>-54</td>
<td>0.90</td>
<td>1.86</td>
<td>0.485</td>
</tr>
<tr>
<td>Zielinski et al. 14</td>
<td>Ni_{50}Si_{15}B_{7}</td>
<td>1.59</td>
<td>-53</td>
<td>0.90</td>
<td>1.93</td>
<td>0.464</td>
</tr>
</tbody>
</table>

In essence, the strength \( \sigma_0 \) represents the critical stress to break a material in mode I cracking; the strength \( \tau_0 \) is the critical resistance to overcome mode II shearing of materials. Thus, the ratio \( \alpha = \tau_0 / \sigma_0 \) can also be regarded as an intrinsic parameter of materials, which affects the failure Modes I or II of different materials.

It is well known that the strengths of various metallic materials are significantly different due to the difference in their microstructures in detail.\[1,2\] Based on the strength analysis above, in principle, it provides a new clue to link the strength relation for a variety of metallic materials with different microstructures from the viewpoint of phenomenology. For ductile single crystals, slip deformation often occurs at a very low critical resolved shear stress \( \tau_0 \) (e.g. \( \sim 1 - 10 \) MPa)\[15\]. Meanwhile, their strength \( \sigma_0 \) should be high enough because cleavage fracture is extremely difficult to occur in ductile single crystals. Therefore, the ratio \( \alpha = \tau_0 / \sigma_0 \) of various ductile single crystals should be very close to 0. According to the Schmid law and the unified tensile criterion, the applied tensile stress for slip deformation can be expressed by

\[
\sigma_T = \tau_0 / \Omega = \tau_0 \sqrt{1 - (\sigma_0 / \sigma_0)^2 / \Omega}. \tag{6}
\]

Here, \( \Omega \) is the Schmid factor of the single crystals, \( \tau_0 \) is the critical resolved shear on the slip plane (typically (111) plane in FCC crystals). With further plastic deformation, the ductile single crystals often display strain hardening, i.e. the critical resolved shear stress \( \tau_0 \) increases owing to the multiplication of dislocations within slip bands\[16\] (see Fig. 3(a)). Besides, there often occur strong interactions between primary and secondary slip bands in the single crystals deformed at high strain level\[15\] as shown in Figure 3(b), which can also cause the obvious increase in the shear strength \( \tau_0 \) due to the lateral hardening mechanism.\[15\] Another important strengthening mechanism is often caused by dislocation piling-up at grain boundaries in bicrystals\[18,19\] or polycrystals,\[20-23\] resulting in a significant increase in the shear strength \( \tau_0 \) due to the GB...
blocking effect to the slip bands, as shown in Figure 3(c). With great grain refinement, shear bands become the predominant plastic deformation mode in ultrafine-grained or nano-crystalline materials, as shown in Figure 3(d). Those materials often possess very high shear strength $\tau_0$ in comparison with their counterparts with coarse grains owing to the refinement strengthening effect. The significant increase in the critical shear strength $\tau_0$ will always lead to a high ratio $\alpha = \tau_0 / \sigma_0$. For metallic glassy materials, it is naturally considered that the amorphous state is the ultimate limit for grain refinement of crystalline materials, and possess the highest ratio $\alpha = \tau_0 / \sigma_0$.

Based on the data available in Table 1, the two intrinsic strengths $\sigma_0$ and $\tau_0$ of different metallic glasses were calculated by substituting their nominal tensile fracture stress $\sigma_T$ and the shear fracture angle $\theta_f$ into Equations (5(a) and 5(b). It can be seen that $\sigma_0$ is in the range of 1.5 – 2.5 GPa for the metallic glasses in Table 1, and $\tau_0$ is in a level of ~ 1.0 GPa, which is greatly higher than the critical resolved shear stress of various single crystals. Therefore, if considering that all the $\sigma_0$ values of the various materials in different states are identical, with increasing shear strength $\tau_0$, the ratio $\alpha = \tau_0 / \sigma_0$ should follow an increasing order from single crystals, bicrystals or coarse-grained, conventional crystalline, ultrafine-grained, nano-crystalline materials and metallic glasses. Assuming that all the materials yield or fail in a shear mode and their strengths follow the unified criterion under tensile loading and Tresca criterion under compressive loading, respectively, their nominal compressive and tensile strengths $\sigma_C$ and $\sigma_T$ can be expressed as,

$$\sigma_C = \tau_0 / (\sin 45^\circ \cos 45^\circ) = 2 \alpha \sigma_0 \quad \text{(Tresca criterion)} \quad (7)$$

and

$$\sigma_T = 2 \alpha \sigma_0 \sqrt{1 - \alpha^2} \quad \text{(Unified criterion)} \quad (8)$$

As illustrated in Figure 4, the dependence of the nominal compressive and tensile strengths ($\sigma_C$ and $\sigma_T$) on the ratio $\alpha = \tau_0 / \sigma_0$ can be clearly seen. When $\alpha = \tau_0 / \sigma_0 \leq 0.240$, the two strengths $\sigma_C$ and $\sigma_T$ are nearly the same, which can well explain why the conventional crystalline materials with coarse grains are seldom to occur the strength asymmetry under compression and tension (see Region A in Figure 4). With further increasing the ratio $\alpha = \tau_0 / \sigma_0$, the strength asymmetry $(\sigma_C - \sigma_T)$ is visible, as marked Region B in Figure 4. Previously, we have summarized that the ratios $\tau_0 / \sigma_0$ of various metallic glasses are in the range of 0.385 – 0.707. Under tensile and compressive loadings, metallic glassy materials often display certain strength asymmetry, which is well consistent with the Region C in Figure 4. The difference strength asymmetry can be attributed to a relatively high value of the ratio $\alpha = \tau_0 / \sigma_0 = 0.385 – 0.707$ according to the unified failure criterion. Between the Regions A and C in Figure 4, the ratio range of $\alpha = \tau_0 / \sigma_0$ is in the range of 0.240 – 0.385, obviously, the strength asymmetry in region B is also slight. It is suggested that the ratio $\alpha = \tau_0 / \sigma_0$ in Region B should correspond to the intrinsic properties of those ultra-fine grained or nano-crystalline materials. In summary, $\sigma_0$ and $\tau_0$ can be regarded as two independent intrinsic strengths for an identical material in nature. The testing nominal fracture strengths ($\sigma_C$ and $\sigma_T$) of various materials do not represent their intrinsic strengths, but are only a reflection of the combined effect of the strengths $\sigma_0$ and $\tau_0$ under different stress states ($\sigma_0$, $\tau_0$). During plastic deformation, all the slip or shear bands of materials are always activated under a combined stress state ($\sigma_0$, $\tau_0$), as illustrated in Figure 3(a) – (d). When the coarse-grained materials are gradually refined to fine-, ultrafine-, or nanocrystaline-grained materials even forming amorphous material, the ratio $\alpha = \tau_0 / \sigma_0$ will be increased continuously. Normally, the nominal compressive strength $\sigma_C$ is mainly determined by the shear strength $\tau_0$ however, the nominal tensile strength $\sigma_T$ is controlled by both of strengths $\sigma_0$ and $\tau_0$. Therefore, when the materials possess a high ratio of $\alpha = \tau_0 / \sigma_0$, it is not difficult to understand why the strength...
asymmetry \((\sigma_C - \tau_f)\) often occurs in those high-strength materials, typically in metallic glassy materials,\(^1\)\(^{14,16-20}\) rocks or ceramics,\(^3\) as well as those ultrafine-grained or nano-crystalline materials.\(^18,20-27\) Furthermore, it is suggested that the proposed new concept about the physical nature of materials strengths is significantly important for better understanding of strengthening mechanisms and optimum design in micro-structures for the high-performance materials practical in engineering application.

**Experimental**

In the current research, different metallic materials were employed, for example pure Cu and Cu-Al single crystals. Cu bicrystals were grown by the Bridgman method in a horizontal furnace.\(^12,13\)\(^{19}\)\(^{20}\)\(^{24}\)\(^{25}\)\(^{26}\)\(^{27}\) ultrafine-grained Al-Cu polycrystals were fabricated by equal channel angular processing (ECAP) technique. Besides, some Zr- and Ti-based bulk metallic glasses were prepared by arc-melting elemental Zr, Cu, Al, Ni and Ti with a purity of 99.9 % or better in a Ti-gettered argon atmosphere.\(^12,13,16\)\(^26\) For reaching homogeneity, the master alloy ingots were re-melted several times and were subsequently cast into copper molds with different dimensions, i.e. 40 mm \(\times\) 30 mm \(\times\) 1.8 mm for tensile test specimens and 3 mm in diameter and a length of 50 mm for the samples used for compressive tests. The compression and the tensile tests of bulk metallic glasses were conducted at different strain rates with an Instron 4466 testing machine at room temperature. Tensile and fatigue tests of the pure Cu and Cu-Al single crystals, Cu bicrystals and ultrafine-grained Al-Cu polycrystals were performed on Shimadzu testing machine. After fracture, all the specimens were investigated by scanning electron microscope (SEM) to reveal the fracture surface morphology and the fracture features.

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